Appendix F

Solutions to Problems in Chapter 6

F.1 Problem 6.1

Short-circuited transmission lines

Section 6.2.1 (book page 193) describes the method to determine the overall length of the transmission line resonator with open-circuited lines. According to our considerations in Chapter 3 a lossless short-circuited transmission line exhibits a reactive input impedance (Equation 3.101, book page 77).

$$Z_{\mathrm{in}} = jZ_0 \tan(\beta\ell) = jX_{\mathrm{in}} \tag{F.1}$$

where ℓ is the length of the transmission line.

In order to design a transmission line resonator we connect two transmission lines in parallel (see Fig. F.1) where ℓ_1 and ℓ_2 are the individual lengths of the lines. The overall length of the resonator is $\ell = \ell_1 + \ell_2$.

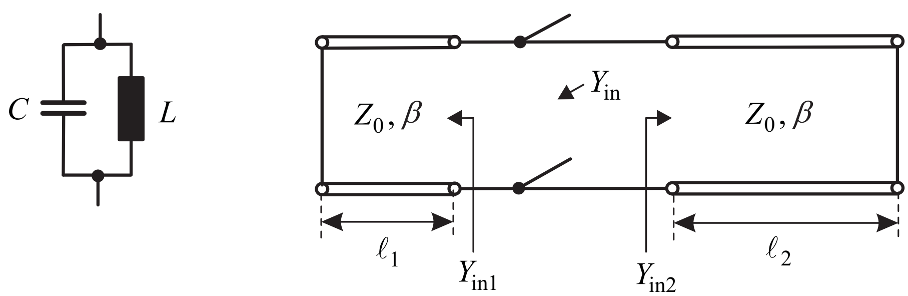

Figure F.1: Transmission line resonator with short-circuited lines

At the input terminal we see the following admittance

$$Y_{\mathrm{in}} = Y_{\mathrm{in1}} + Y_{\mathrm{in2}} = \frac{1}{jZ_0}\left(\frac{1}{\tan(\beta\ell_1)} + \frac{1}{\tan(\beta\ell_2)}\right) = \frac{1}{jZ_0}\left(\cot(\beta\ell_1) + \cot(\beta\ell_2)\right) \tag{F.2}$$

At parallel resonance the imaginary part of the admittance is zero. Therefore, we rewrite the sum of cotangent functions as

$$\cot x \pm \cot y = \pm\frac{\sin(x \pm y)}{\sin x \sin y} \tag{F.3}$$

We get

$$\mathrm{Im}\{Y_{\mathrm{in}}\} = -\frac{1}{Z_0} \cdot \frac{\sin(\beta(\ell_1+\ell_2))}{\sin(\beta\ell_1)\sin(\beta\ell_2)} = 0 \qquad \text{(at resonance)} \tag{F.4}$$

The sine function in the numerator determines the zeros of the expression.

$$\beta(\ell_1+\ell_2) = n\pi \qquad \text{where} \qquad n \in \mathbb{Z} \tag{F.5}$$

Therefore, the overall length of the resonator is

$$\boxed{\ell = \ell_1+\ell_2 = \frac{n\pi}{\beta} = \frac{n\pi\lambda}{2\pi} = n\frac{\lambda}{2}} \tag{F.6}$$

A transmission line with short-circuited terminations at both ends shows parallel resonance for a line length equal to a half wavelength.

Resonator with a short-circuited and an open-circuited line

Let us consider the configuration in Figure F.2.

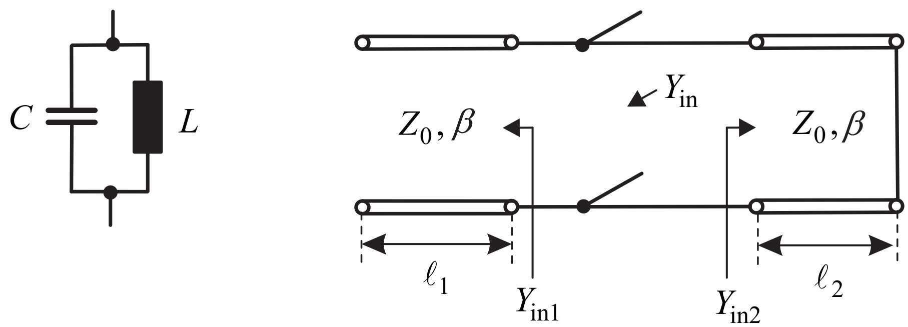

Figure F.2: Transmission line resonator with short-circuited and open-circuited line

The input impedance of the open-circuited line is

$$Z_{\mathrm{in1}} = -jZ_0\cot(\beta\ell_1) = jX_{\mathrm{in1}} \tag{F.7}$$

The input impedance of the short-circuited line is

$$Z_{\mathrm{in2}} = jZ_0\tan(\beta\ell_2) = jX_{\mathrm{in2}} \tag{F.8}$$

The admittance at the input terminal becomes

$$Y_{\mathrm{in}} = Y_{\mathrm{in1}} + Y_{\mathrm{in2}} = \frac{1}{jZ_0}\left(-\frac{1}{\cot(\beta\ell_1)} + \frac{1}{\tan(\beta\ell_2)}\right) = \frac{1}{jZ_0}\left(\cot(\beta\ell_2) - \tan(\beta\ell_1)\right) \tag{F.9}$$

At parallel resonance the imaginary part of the admittance is zero. Therefore, we rewrite the expression in the numerator as

$$\cot x - \tan y = \frac{\cos(x+y)}{\sin x \cos y} \tag{F.10}$$

and get the following condition

$$\operatorname{Im}\{Y_{\text{in}}\} = -\frac{1}{Z_0} \cdot \frac{\cos(\beta(\ell_1 + \ell_2))}{\sin(\beta\ell_2)\sin(\beta\ell_1)} = 0 \qquad \text{(at resonance)} \tag{F.11}$$

We find the first zero of the cosine function at $\pi/2$.

$$\beta(\ell_1 + \ell_2) = \frac{\pi}{2} \qquad \rightarrow \qquad \ell = \ell_1 + \ell_2 = \frac{\pi}{2} \cdot \frac{1}{\beta} = \frac{\pi}{2} \cdot \frac{\lambda}{2\pi} = \frac{\lambda}{4} \tag{F.12}$$

Therefore, the shortest line length ℓ is a quarter of a wavelength.

F.2 Problem 6.2

a) Resistive termination $330\,\Omega$

A resistive load termination $Z_{\text{A1}} = 330\,\Omega$ shall be matched to a reference impedance of $Z_{\text{in}} = 50\,\Omega$ by a LC-network at a frequency $f = 900\,\text{MHz}$. According to Section 6.3.1 low-pass and high-pass realisations exist (book page 197).

Figure F.3 shows the corresponding circuits and Smith chart transformations using ADS circuit simulator from Agilent. The Smith diagram shows red circles (admittances) and blue circles (impedances). The matching point (zero reflection coefficient) is located at the center of the diagram. The transformation paths (at $f = 900\,\text{MHz}$) are show by coloured arrows. Furthermore, Figure F.3 shows the magnitude of the reflection coefficient in the frequency range from DC to $f = 1800\,\text{MHz}$. As expected the reflection coefficient is practically zero at $f = 900\,\text{MHz}$.

For comparison we calculate the component values using the formulas given in Section 6.3.1. For $R_{\text{A}} > R_{\text{I}}$ we get

$$|X_1| = R_{\text{I}} \cdot Q = 118.3\,\Omega \quad \text{and}$$

$$|X_2| = R_{\text{A}}/Q = 139.5\,\Omega \quad \text{where} \quad Q = \sqrt{\frac{R_{\text{A}}}{R_{\text{I}}} - 1} = 2.366 \tag{F.13}$$

The reactances X_1 and X_2 represent reactive components, that are either inductive ($X = \omega L$) or capacitive $X = -1/(\omega C)$. High-pass and low-pass configurations consist of two different components: one is a capacitance and one is an inductance (Figure 6.12). The component values for a given frequency f_0 are determined by the following equations and summarized in Table F.1. The results are in good agreement. Minor differences are caused by the graphical evaluation of the Smith chart.

$$L_{1,2} = \frac{|X_{1,2}|}{2\pi f_0} \qquad \text{and} \qquad C_{1,2} = \frac{1}{|X_{1,2}| 2\pi f_0} \tag{F.14}$$

	Series component	Shunt component
Low-pass design	$L_1 = 20.92\,\text{nH}$ ($20.82\,\text{nH}$)	$C_2 = 1.268\,\text{pF}$ ($1.277\,\text{pF}$)
High-pass design	$C_1 = 1.495\,\text{pF}$ ($1.498\,\text{pF}$)	$L_2 = 24.67\,\text{nH}$ ($24.58\,\text{nH}$)

Table F.1: Component values for $Z_{\text{A}} = 330\,\Omega$ (Smith chart results in parentheses)

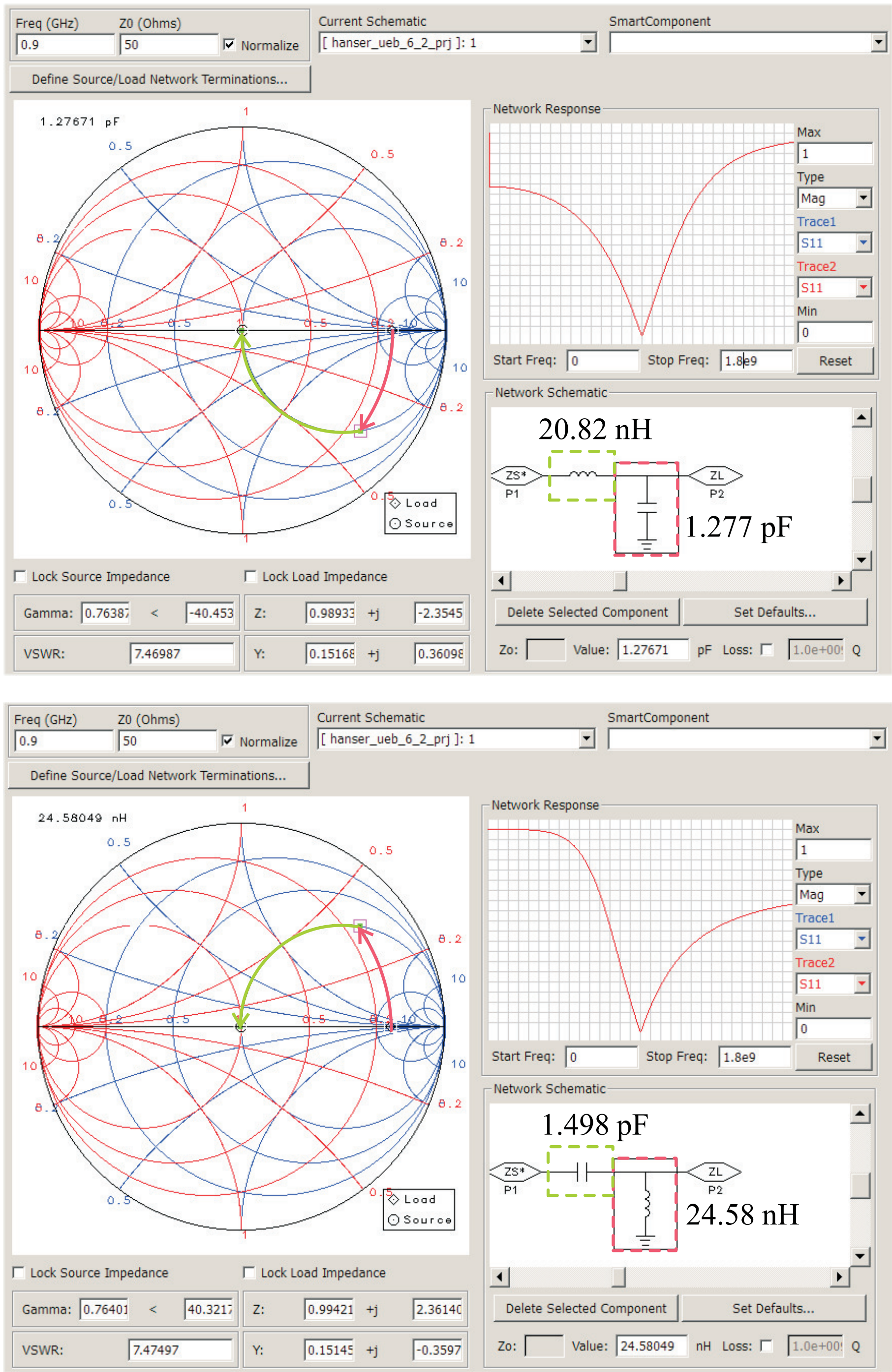

Figure F.3: Solution to Problem 6.2a (low-pass (top), high-pass (bottom))

b) Resistive termination $10\,\Omega$

We follow the method in the previous paragraph and use equations from Section 6.3.1. Now we have $R_\mathrm{A} < R_\mathrm{I}$. So we get

$$\begin{aligned} |X_1| &= R_\mathrm{I}/Q = 25\,\Omega \quad \text{and} \\ |X_2| &= R_\mathrm{A} \cdot Q = 20\,\Omega \quad \text{where} \quad Q = \sqrt{\frac{R_\mathrm{I}}{R_\mathrm{A}} - 1} = 2 \end{aligned} \tag{F.15}$$

Component values are given as

$$L_{1,2} = \frac{|X_{1,2}|}{2\pi f_0} \qquad \text{and} \qquad C_{1,2} = \frac{1}{|X_{1,2}| 2\pi f_0} \tag{F.16}$$

Table F.2 lists the values for high-pass and low-pass design. Figure F.4 shows the design using a Smith chart tool.

	Series component	Shunt component
High-pass design	$L_1 = 4.42\,\mathrm{nH}$ ($4.44\,\mathrm{nH}$)	$C_2 = 8.84\,\mathrm{pF}$ ($8.84\,\mathrm{pF}$)
Low-pass design	$C_1 = 7.07\,\mathrm{pF}$ ($7.12\,\mathrm{pF}$)	$L_2 = 3.54\,\mathrm{nH}$ ($3.53\,\mathrm{nH}$)

Table F.2: Component values for $Z_\mathrm{A} = 10\,\Omega$ (Smith chart results in parentheses)

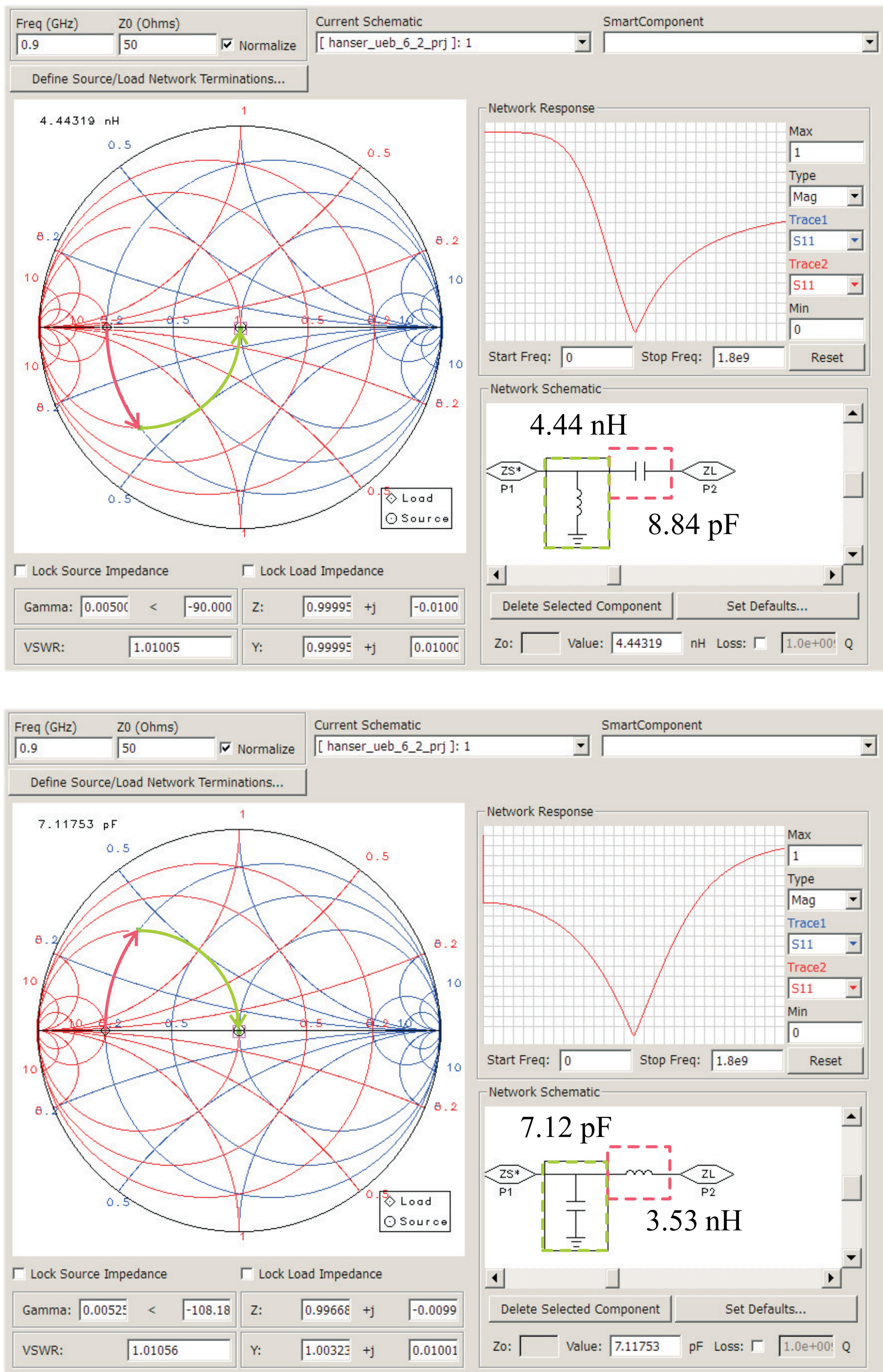

Figure F.4: Solution to Problem 6.2b

c) Complex termination $(200 + j100)\,\Omega$

Figure F.5 shows the transformation paths in the Smith diagram.

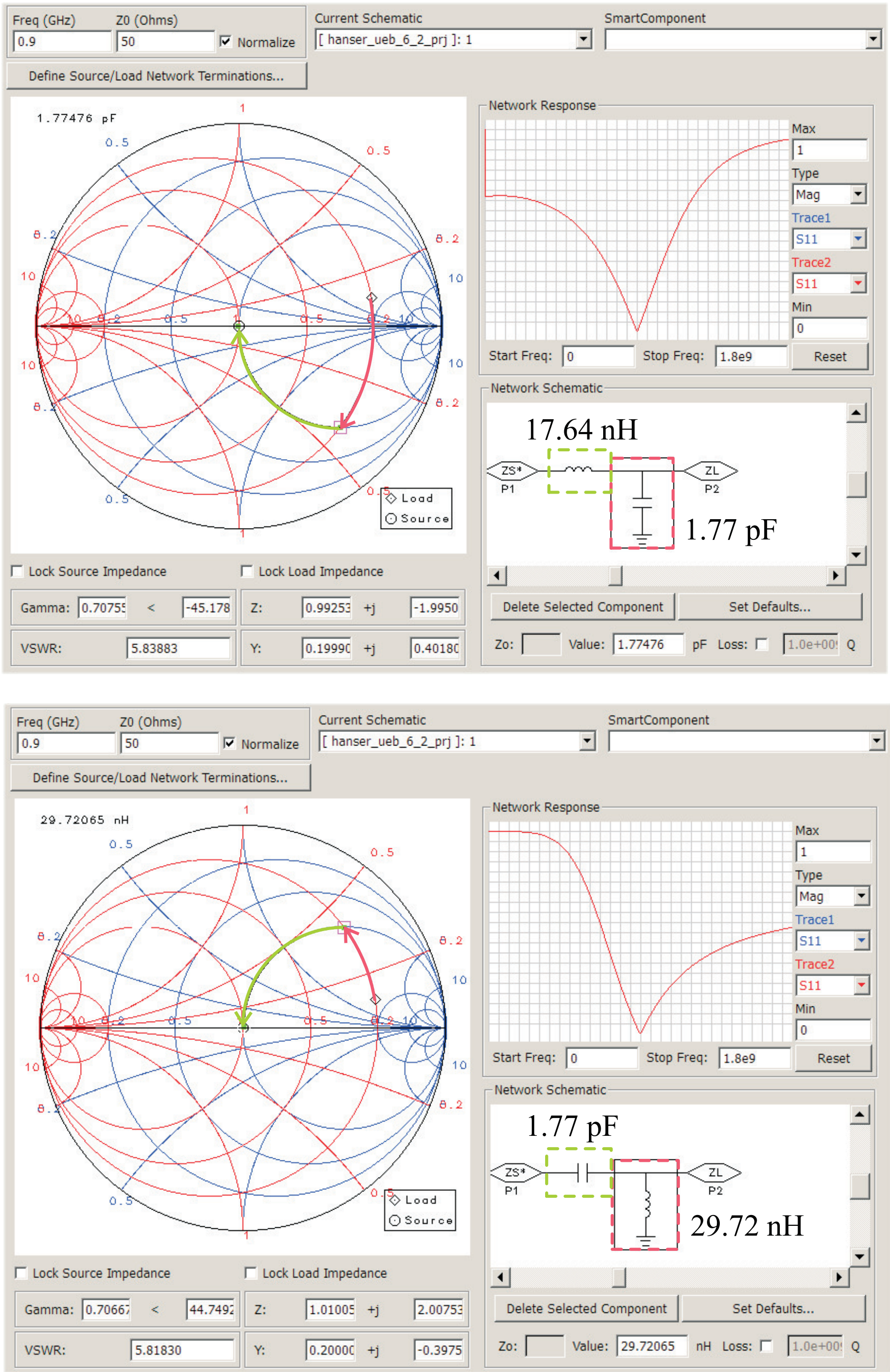

Figure F.5: Solution to Problem 6.2c

d) Complex termination $(15 - j75)\,\Omega$

Figure F.6 shows the transformation paths in the Smith diagram.

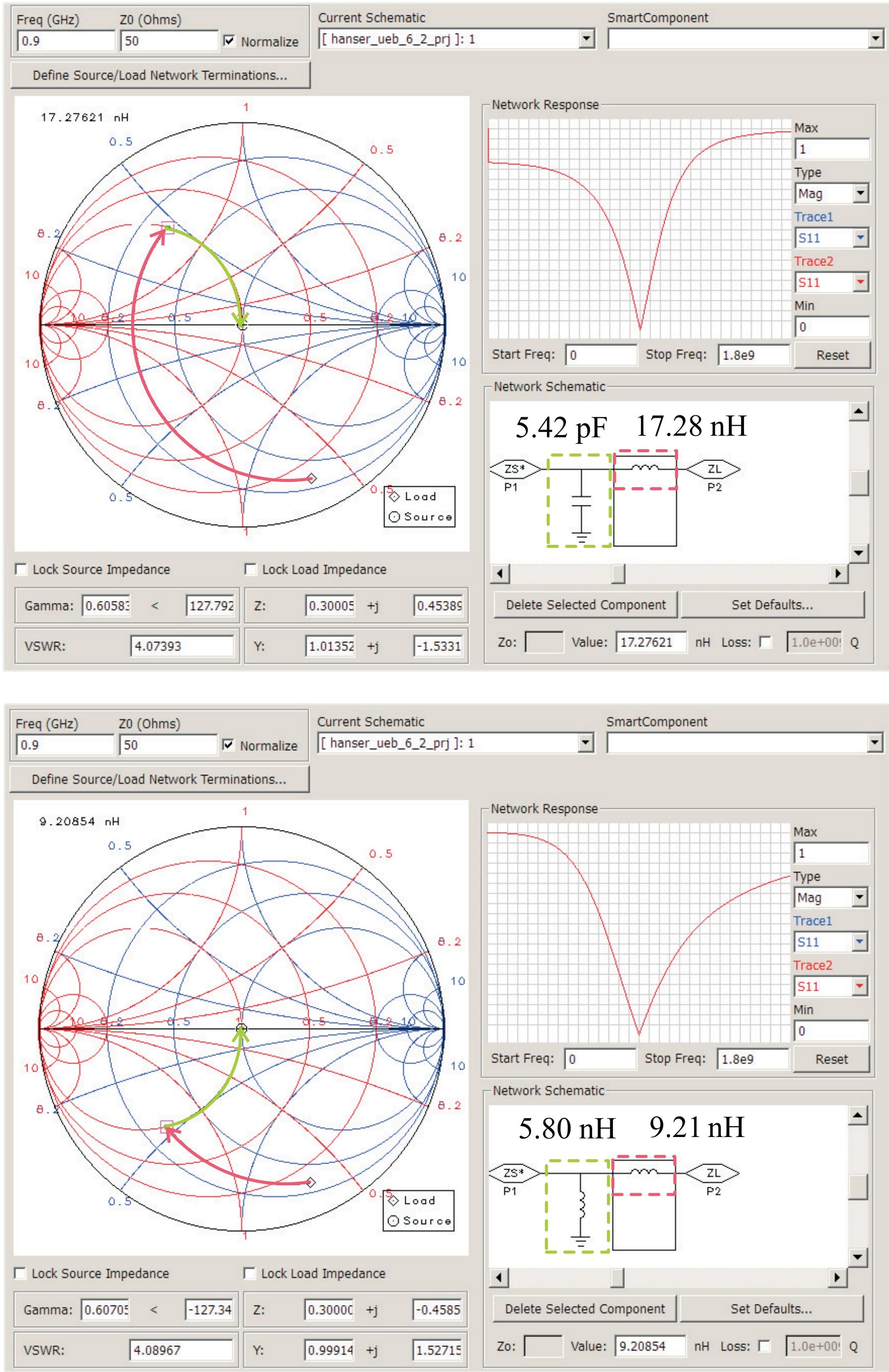

Figure F.6: Solution to Problem 6.2d

F.3 Problem 6.3

A load resistance of $R_\mathrm{A} = 1100\,\Omega$ shall be matched to a reference impedance of $R_\mathrm{I} = 50\,\Omega$ at a frequency of $f = 1\,\mathrm{GHz}$. We will investigate the bandwidth of single-stage, two-stage and three-stage networks.

1. Single-stage LC-matching network

We determine the component values by using the equations of Section 6.3.1.

$$Q = \sqrt{\frac{R_\mathrm{A}}{R_\mathrm{I}} - 1} = 4.5826 \tag{F.17}$$

$$X_1 = R_\mathrm{I} \cdot Q = 229.13\,\Omega \qquad \text{and} \qquad X_2 = R_\mathrm{A}/Q = 240.04\,\Omega \tag{F.18}$$

We choose a high-pass design and get

$$C_1 = \frac{1}{\omega X_1} = 0.69\,\mathrm{pF} \qquad \text{and} \qquad L_2 = \frac{X_2}{\omega} = 38.2\,\mathrm{nH} \tag{F.19}$$

Figure F.7 shows the schematic (ADS) and Figure F.8 illustrates the transformation path in the Smith chart. The reflection coefficient in the frequency range from 0.9 GHz to 1.1 GHz is given in Figure F.9 (red curve).

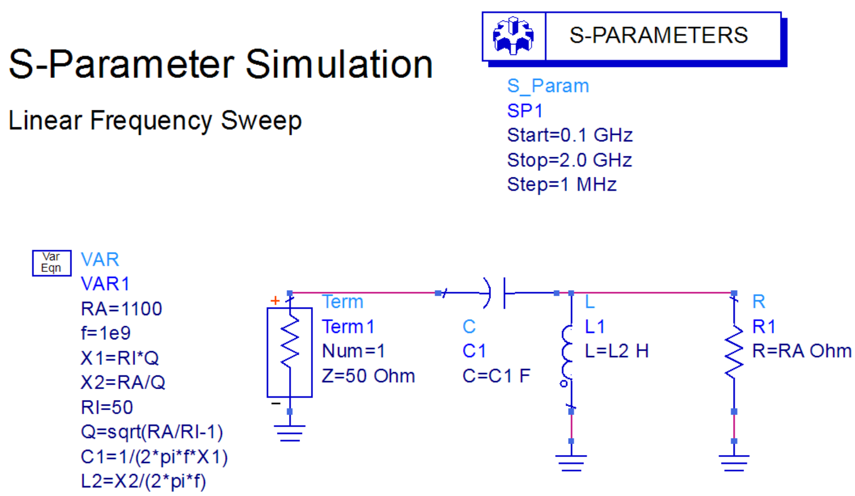

Figure F.7: Single-stage LC-matching network (high-pass design)

2. Two-stage LC-matching network

Now matching is achieved in two steps. First, we transform the load impedance to an intermediate resistance value

$$R_\mathrm{m} = \sqrt{R_\mathrm{A} R_\mathrm{I}} \tag{F.20}$$

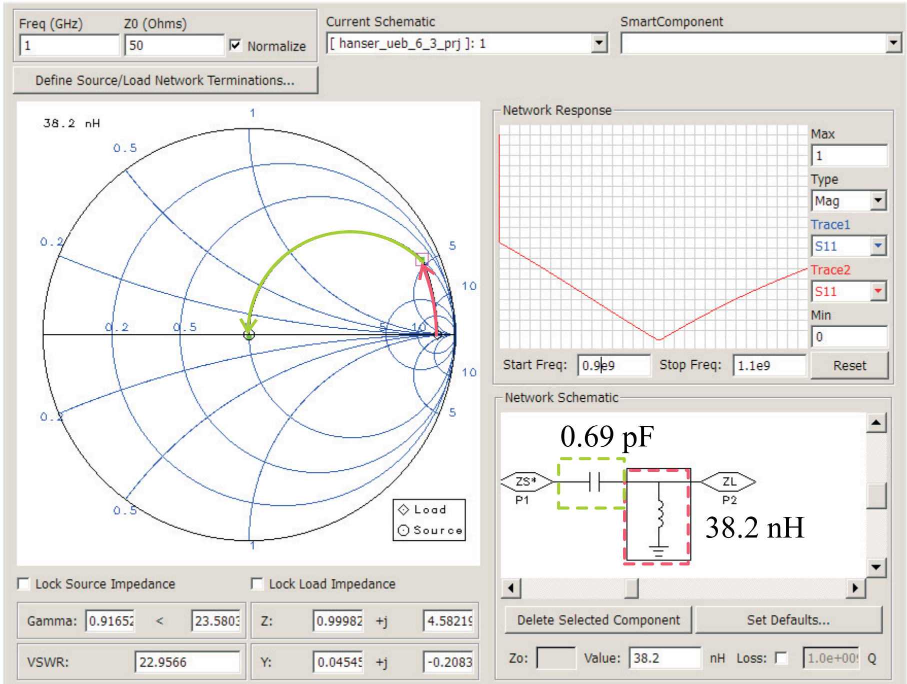

Figure F.8: Smith chart for single LC-matching network (high-pass design)

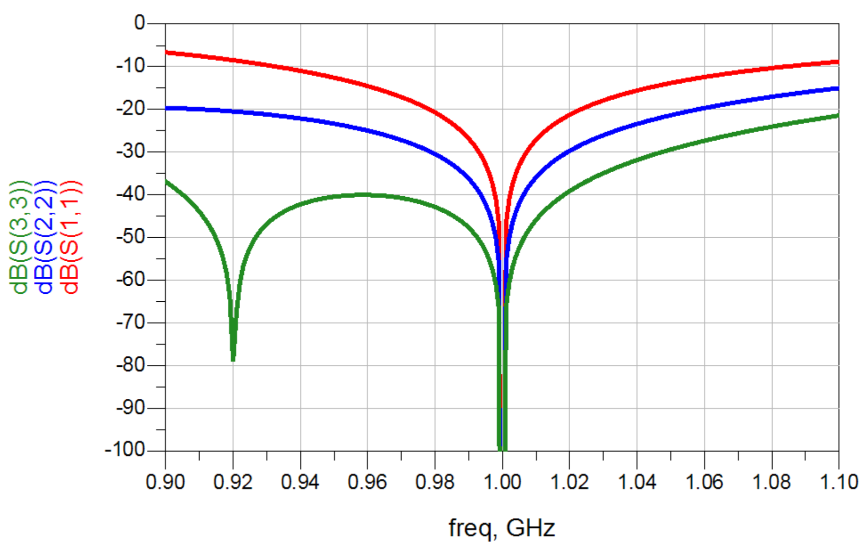

Figure F.9: Scattering parameters for single LC-matching network (red), two-stage network (blue) and three-stage network (green)

Using our formulas from Section 6.3.1 the component values of the two LC-networks ($R_\mathrm{A} \to R_\mathrm{m}$ and $R_\mathrm{m} \to R_\mathrm{I}$) are

$$Q = Q_1 = \sqrt{\frac{R_\mathrm{m}}{R_\mathrm{I}} - 1} = Q_2 = \sqrt{\frac{R_\mathrm{A}}{R_\mathrm{m}} - 1} = 1.921 \tag{F.21}$$

$$X_{11} = R_\mathrm{I} \cdot Q = 96.05\,\Omega \qquad \text{and} \qquad X_{21} = R_\mathrm{m} \cdot Q = 450.51\,\Omega \tag{F.22}$$

$$X_{12} = R_\mathrm{m}/Q = 122.08\,\Omega \qquad \text{and} \qquad X_{22} = R_\mathrm{m}/Q = 572.6\,\Omega \tag{F.23}$$

$$\tag{F.24}$$

Choosing a high-pass design we get

$$C_{11} = \frac{1}{\omega X_{11}} = 1.657\,\mathrm{pF} \qquad \text{and} \qquad L_{12} = \frac{X_{12}}{\omega} = 19.4\,\mathrm{nH} \tag{F.25}$$

$$C_{21} = \frac{1}{\omega X_{21}} = 0.35\,\mathrm{pF} \qquad \text{and} \qquad L_{22} = \frac{X_{22}}{\omega} = 91.0\,\mathrm{nH} \tag{F.26}$$

Figure F.10 shows the schematic (ADS) and Figure F.11 illustrates the transformation path in the Smith chart. The reflection coefficient in the frequency range from 0.9 GHz to 1.1 GHz is given in Figure F.9 (blue curve). The two-stage network shows an extended bandwidth.

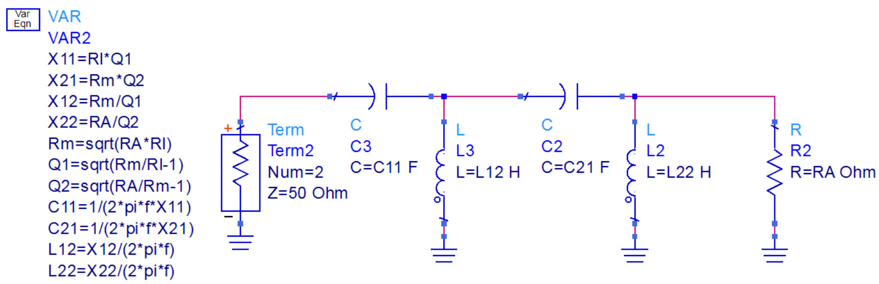

Figure F.10: Two-stage LC-matching network (high-pass design)

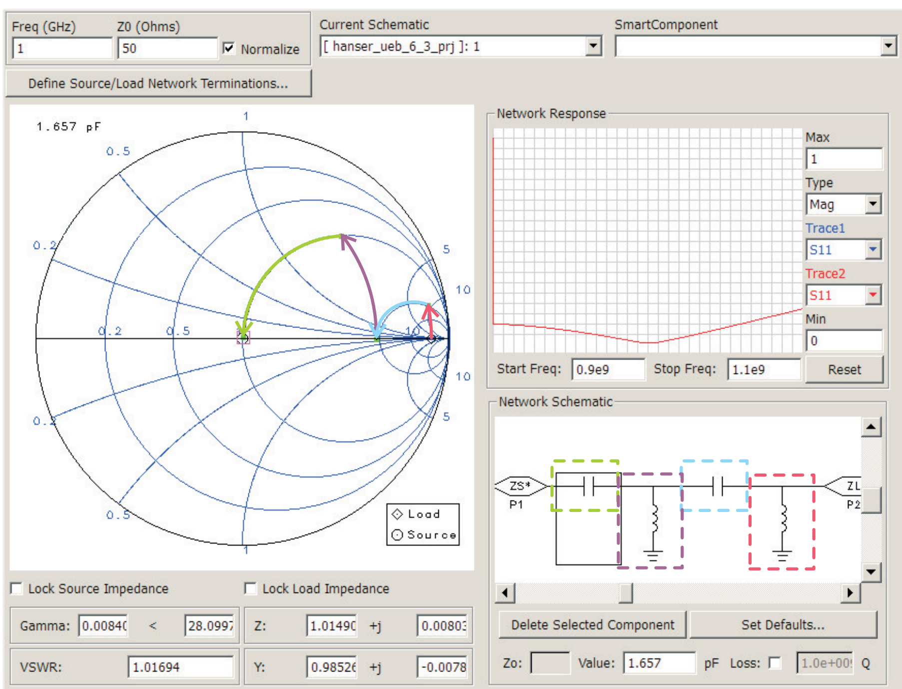

Figure F.11: Smith chart for a two-stage LC-matching network (high-pass design)

3. Three-stage LC-matching network

Now matching is achieved in three steps. We transform to intermediate resistance values of

$$R_{\mathrm{mo}} = \sqrt{R_{\mathrm{A}}^2 R_{\mathrm{I}}} \qquad \text{and} \qquad R_{\mathrm{mu}} = \sqrt{R_{\mathrm{A}} R_{\mathrm{I}}^2} \tag{F.27}$$

Using our formulas from Section 6.3.1 the component values of the three LC-networks ($R_{\mathrm{A}} \to R_{\mathrm{mo}}$ and $R_{\mathrm{mo}} \to R_{\mathrm{mu}}$ and $R_{\mathrm{mu}} \to R_{\mathrm{I}}$) are

$$Q = Q_1 = \sqrt{\frac{R_{\mathrm{mu}}}{R_{\mathrm{I}}} - 1} = Q_2 = \sqrt{\frac{R_{\mathrm{mo}}}{R_{\mathrm{mu}}} - 1} = Q_3 = \sqrt{\frac{R_{\mathrm{A}}}{R_{\mathrm{mo}}} - 1} = 1.342 \tag{F.28}$$

$$X_{011} = R_{\mathrm{I}} \cdot Q = 67.1\,\Omega \quad ; \quad X_{021} = R_{\mathrm{mu}} \cdot Q = 188.0\,\Omega \quad ; \quad X_{031} = R_{\mathrm{mo}} \cdot Q = 526.8\,\Omega \tag{F.29}$$

$$X_{012} = R_{\mathrm{mu}}/Q = 105.82\,\Omega \quad ; \quad X_{022} = R_{\mathrm{mo}}/Q = 292.53\,\Omega \quad ; \quad X_{032} = R_{\mathrm{A}}/Q = 819.67\,\Omega \tag{F.30}$$

Choosing a high-pass design we get

$$C_{011} = \frac{1}{\omega X_{011}} = 2.37\,\mathrm{pF} \qquad \text{and} \qquad L_{012} = \frac{X_{012}}{\omega} = 16.8\,\mathrm{nH} \tag{F.31}$$

$$C_{021} = \frac{1}{\omega X_{021}} = 0.847\,\mathrm{pF} \qquad \text{and} \qquad L_{022} = \frac{X_{022}}{\omega} = 46.6\,\mathrm{nH} \tag{F.32}$$

$$C_{031} = \frac{1}{\omega X_{031}} = 0.302\,\mathrm{pF} \qquad \text{and} \qquad L_{032} = \frac{X_{032}}{\omega} = 130.4\,\mathrm{nH} \tag{F.33}$$

Figure F.12 shows the schematic (ADS) and Figure F.13 illustrates the transformation path in the Smith chart. The reflection coefficient in the frequency range from 0.9 GHz to 1.1 GHz is given in Figure F.9 (green curve). The three-stage network shows an extended bandwidth in comparison to the two-stage network.

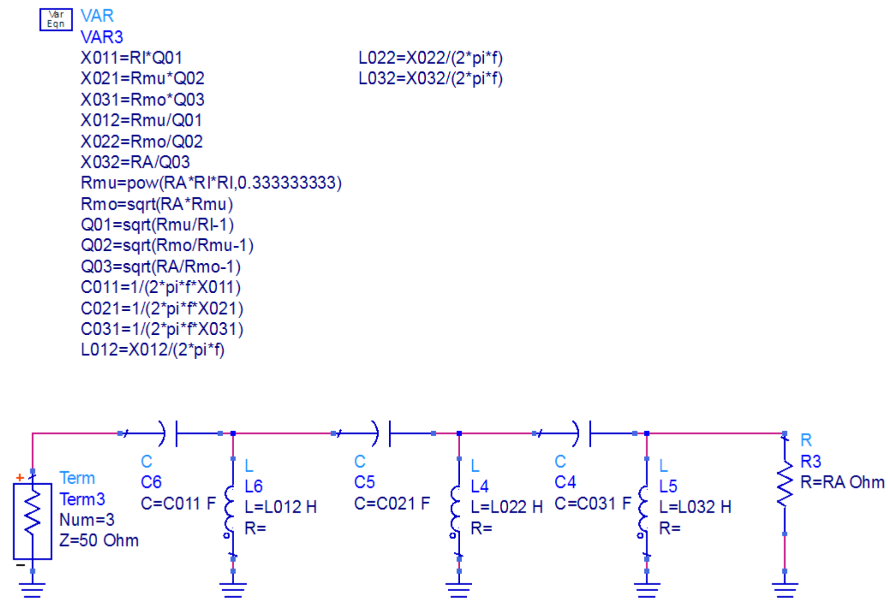

Figure F.12: Three-stage LC-matching network (high-pass design)

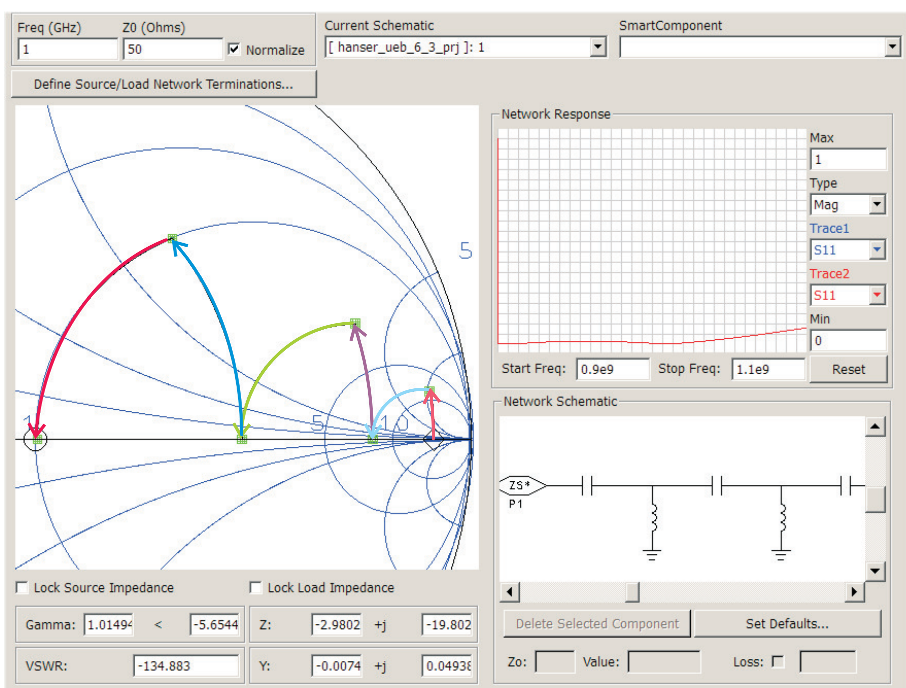

Figure F.13: Smith chart for three-stage LC-matching network (high-pass design)

F.4 Problem 6.4

In Section 6.3.2.1. (book page 200) we introduced a simple microstrip power divider (substrate material: alumina; relative permittivity $\varepsilon_r = 9.8$; substrate height $h = 635\,\mu$m). A quarter-wave transformer is used to match port 1 to $50\,\Omega$ at a frequency of $f = 5\,$GHz. In the following we design and compare different circuits.

1. Single-stage quarter-wave transformer

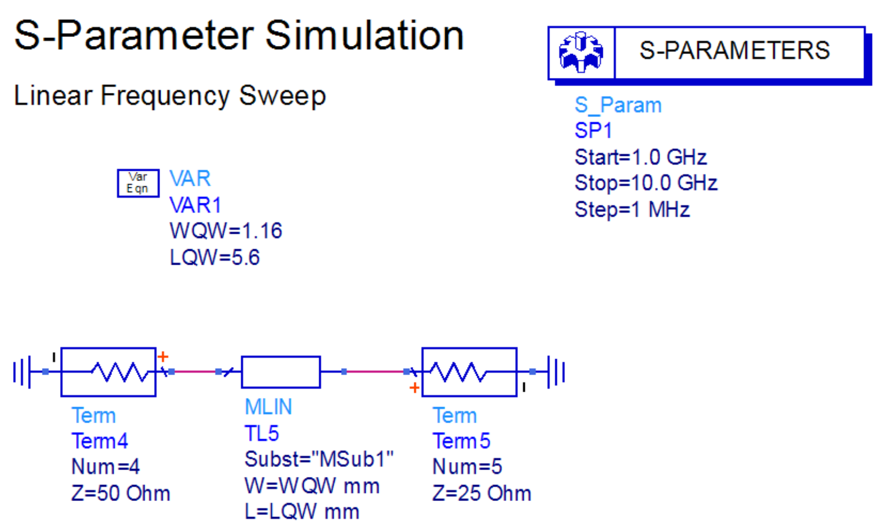

Figure F.14: Quarter-wave transformer ($25\,\Omega \rightarrow 50\,\Omega$)

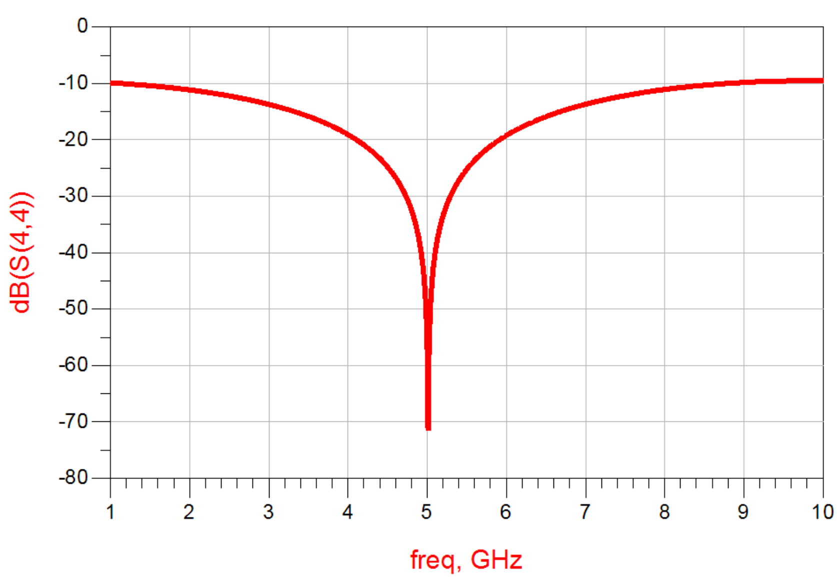

Figure F.15: Reflection coefficient at port 1 for the quarter-wave transformer ($25\,\Omega \rightarrow 50\,\Omega$)

The parallel circuit of the two outgoing lines (Figure 6.14) has an impedance of 25 Ω. Therefore, the line impedance of the quarter-wave transformer is

$$Z_0 = \sqrt{50\,\Omega\, 25\,\Omega} = 35.4\,\Omega \tag{F.34}$$

Using a transmission line calculator (e.g. TX-Line from AWR or LineCalc from Agilent) gives us the following microstrip line dimensions

$$L = 5.6\,\text{mm} \qquad \text{and} \qquad W = 1.16\,\text{mm} \tag{F.35}$$

Figure F.14 shows the schematic. The reflection coefficient is given in Figure F.15.

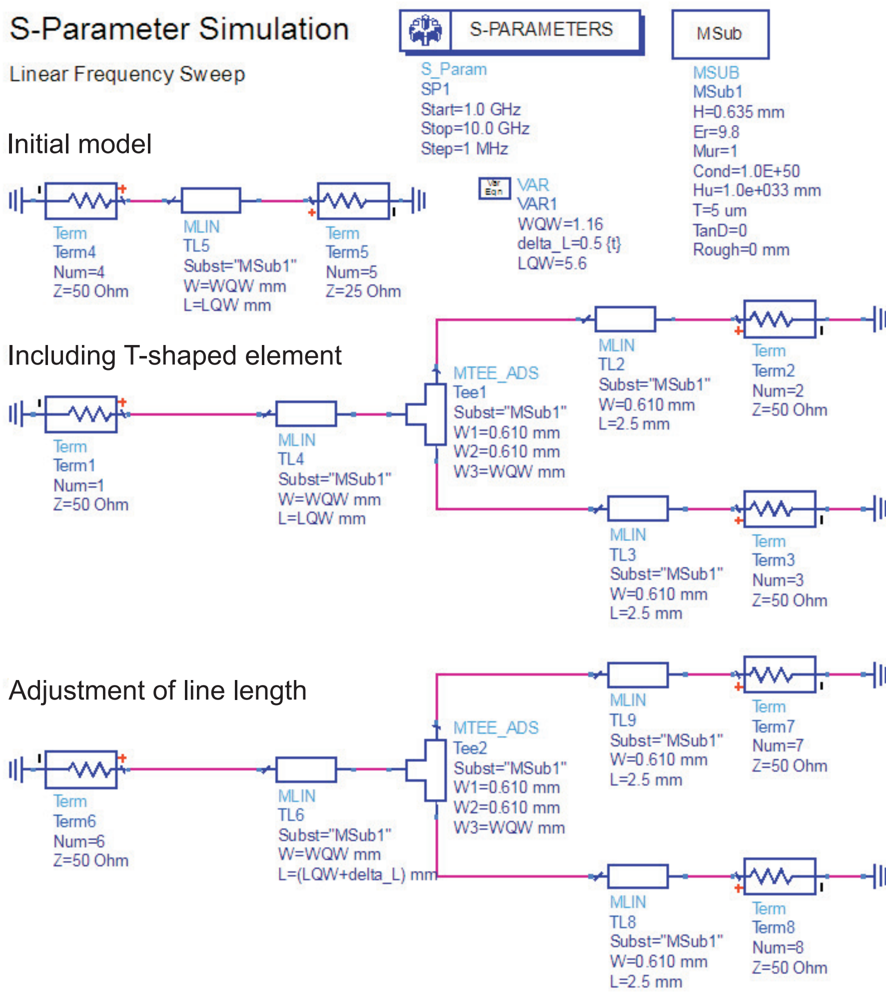

Figure F.16: Adjusting line length to compensate for T-shaped element effect

In our example (Section 6.3.2.1) the outgoing lines are connected by a T-shaped element (a rectangular metallic plate that connects outgoing lines and quarter-wave transformer). This T-shaped element alters the impedance slightly, so that we no longer see exactly the expected impedance of 25 Ω at the end of the quarter-wave transformer. By adjusting the length of the quarter-wave transformer matching is restored. Figure F.16 shows the schematics. The reflection coefficients are given in Figure F.17.

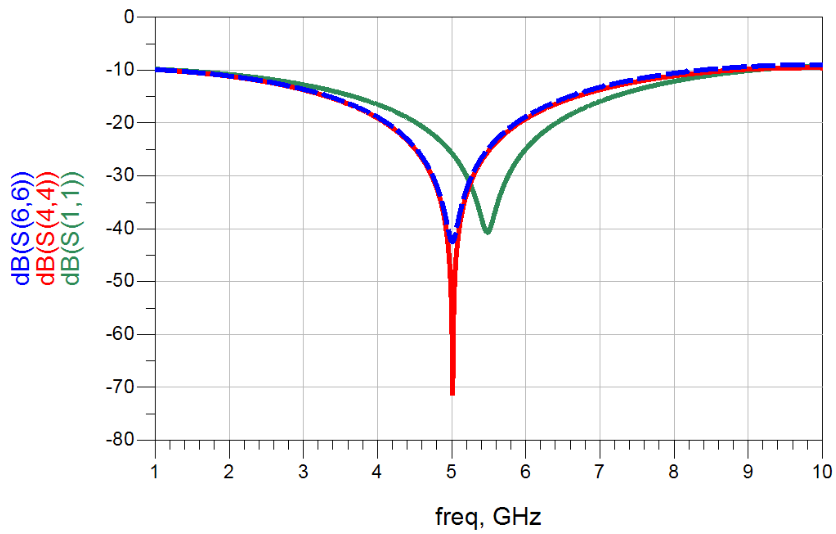

Figure F.17: Reflection coefficients: Initial design ($25\,\Omega \rightarrow 50\,\Omega$) (red); after introducing real microstrip lines and T-shaped element (green); adjusting line length to compensate for T-shaped element effect (blue)

2. Quarter-wave transformer (two-stage design)

Now we will investigate the effect of a two-stage design on the resulting bandwidth. As in Problem 6.3 we transform in a first step to an intermediate resistance.

$$R_{\mathrm{m}} = \sqrt{50\,\Omega\, 25\,\Omega} = 35.4\,\Omega \tag{F.36}$$

The line impedances of the quarter-wave transformers are

$$Z_{01} = \sqrt{50\,\Omega\, R_{\mathrm{m}}} = 42.1\,\Omega \qquad \text{and} \qquad Z_{02} = \sqrt{R_{\mathrm{m}}\, 25\,\Omega} = 29.7\,\Omega \tag{F.37}$$

By using a transmission line calculator the microstrip dimensions become

$$L_1 = 5.7\,\mathrm{mm} \qquad \text{and} \qquad W_1 = 0.86\,\mathrm{mm} \tag{F.38}$$
$$L_2 = 5.51\,\mathrm{mm} \qquad \text{and} \qquad W_2 = 1.54\,\mathrm{mm} \tag{F.39}$$

The schematic is given in Figure F.18. The reflection coefficient is shown in Figure F.19 and compared to the single-stage design. The two-stage design has larger bandwidth.

Finally, we will investigate the effect of the T-shaped element in our microstrip design. This can be easily done using a circuit simulator (Figure F.20). The reflection coefficients are shown in Figure F.21.

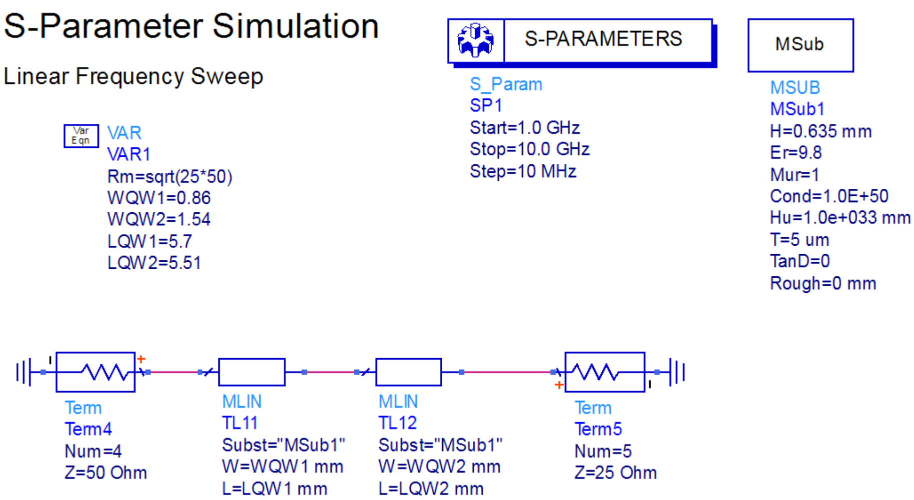

Figure F.18: Two-stage quarter-wave transformer

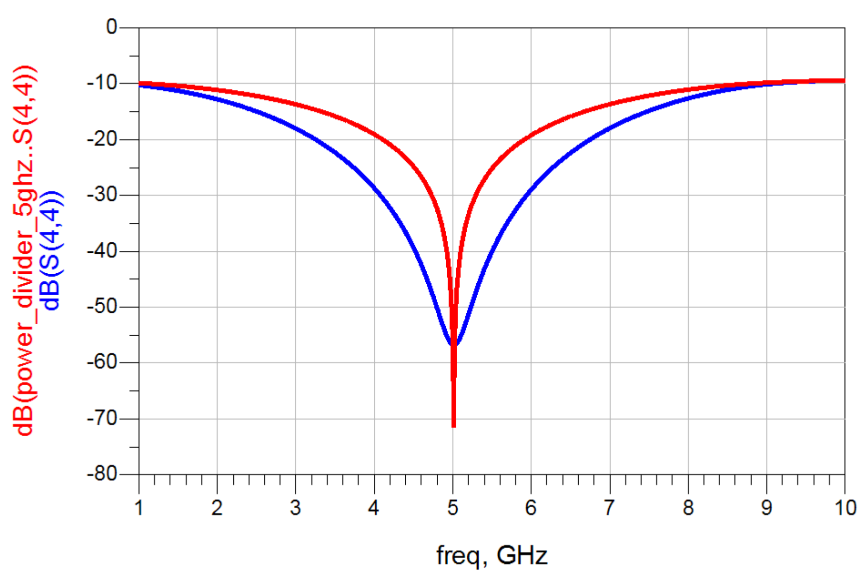

Figure F.19: Reflection coefficients of single-stage (red) and two-stage (blue) quarter-wave transformer (load resistance $25\,\Omega$)

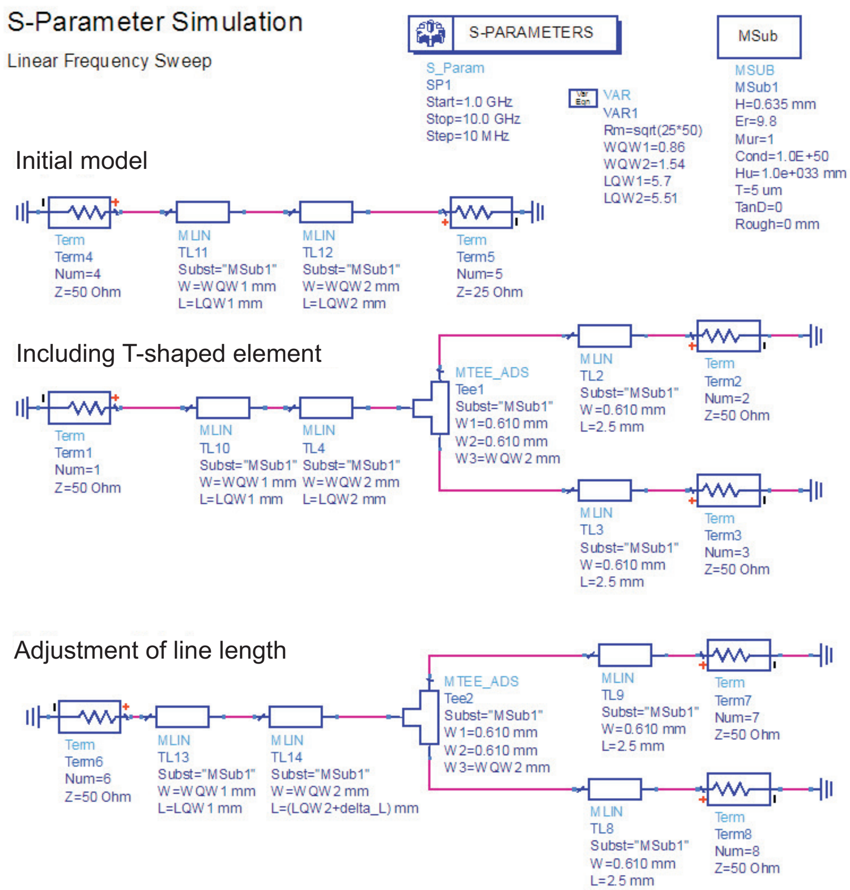

Figure F.20: Investigating and compensating the effect of the T-shaped element

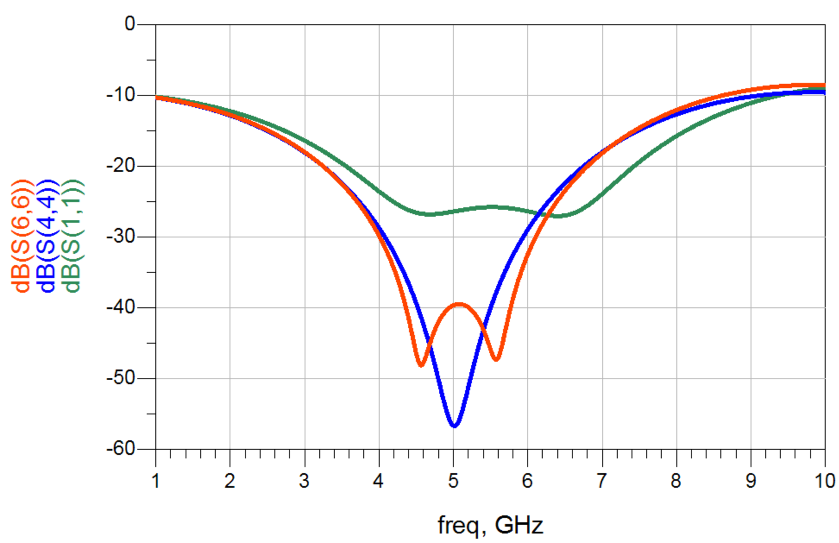

Figure F.21: Reflection coefficients for the two-stage-design: with ideal load of $25\,\Omega$ (blue); with outgoing $50\,\Omega$-lines and T-element as load (green); with outgoing $50\,\Omega$-lines and T-element as load (red) but adjusted length of quarter-wave transformer

F.5 Problem 6.5

Figure F.22 shows the design of the matching circuit using the Smith chart tool in ADS. The resulting electric line length is 131.2° for the serial line and 37.2° for the stub line. The dimensions of the microstrip lines are determined by TX-Line. Figure F.23 shows the graphical user interface and the dimensions of the lines.

$$w_\mathrm{s} = 0.3\,\mathrm{mm} \quad \text{and} \quad \ell_\mathrm{s} = 4.69\,\mathrm{mm} \quad \text{(serial line)} \tag{F.40}$$

$$w_\mathrm{p} = 0.3\,\mathrm{mm} \quad \text{and} \quad \ell_\mathrm{p} = 1.33\,\mathrm{mm} \quad \text{(stub (parallel) line)} \tag{F.41}$$

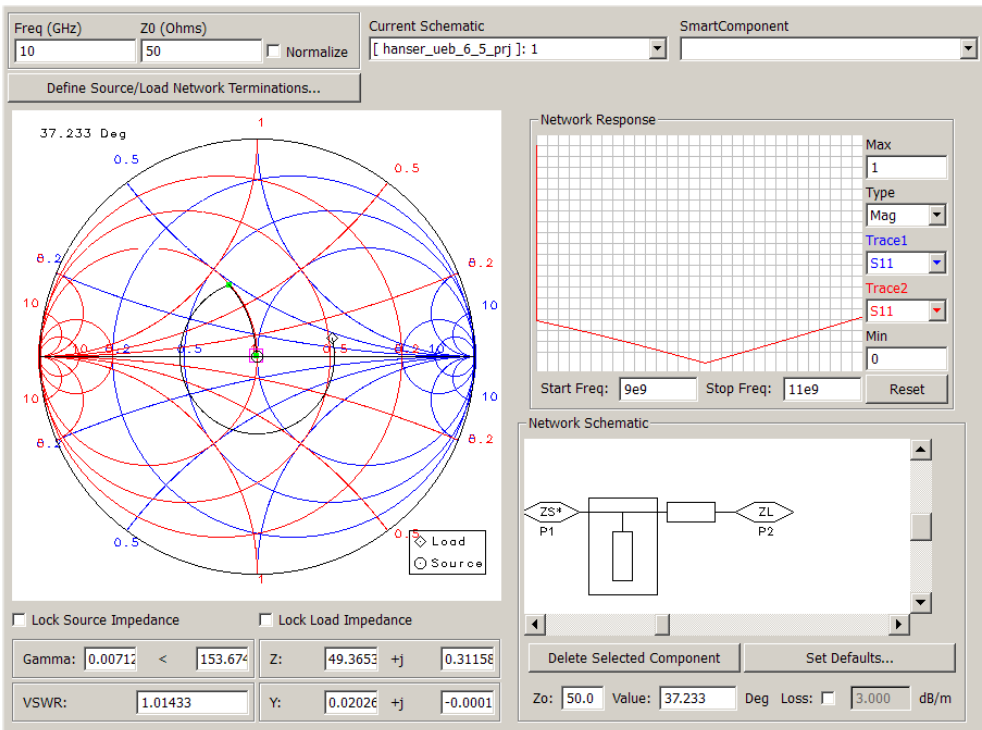

Figure F.22: Design using Smith chart tool in ADS

In order to validate our results we set up a circuit simulation (Figure F.24). First, we use *ideal* lines. TL4 represents a serial line and TL5 an open ended stub line in parallel. The open end is modelled by a very high resistance value ($1\,\mathrm{T\Omega}$). The green curve in Figure F.25 shows the resulting reflection coefficient with matching at 10 GHz.

Next, we consider microstrip lines. The blue curve in Figure F.25 shows that matching is off the intended frequency. Our initial model did not take into account

- capacitive open end effect of the stub line and
- influence of the connecting element (MTEE), that has to be included for geometrical reasons.

By tuning the line lengths manually we achieve matching at 10 GHz (red curve). The geometry of the final design is shown in Figure F.26.

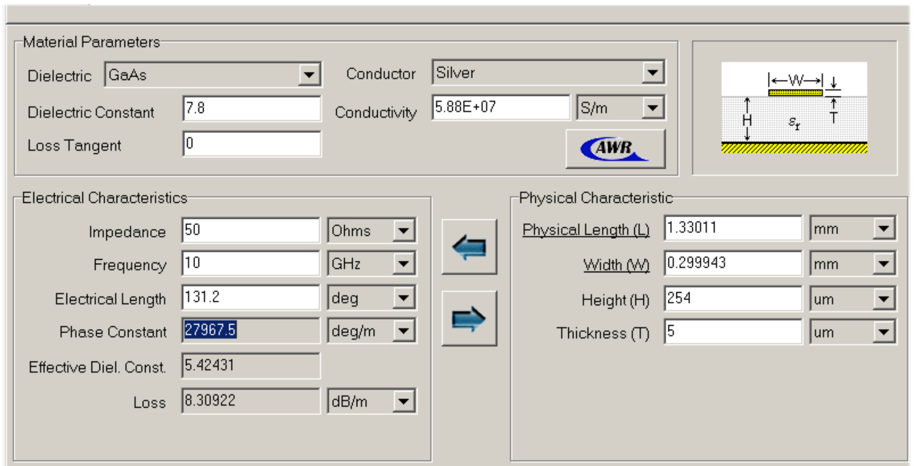

Figure F.23: Design of microstrip lines using TX-Line

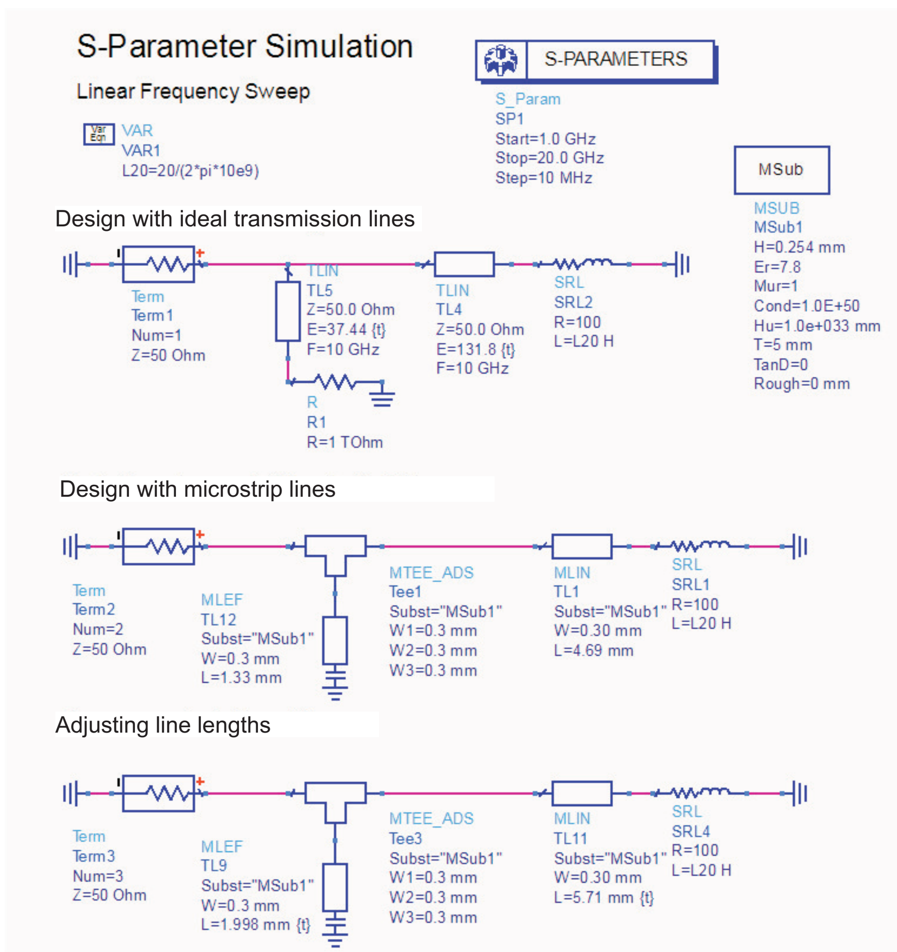

Figure F.24: Circuit simulation with ideal lines and microstrip lines

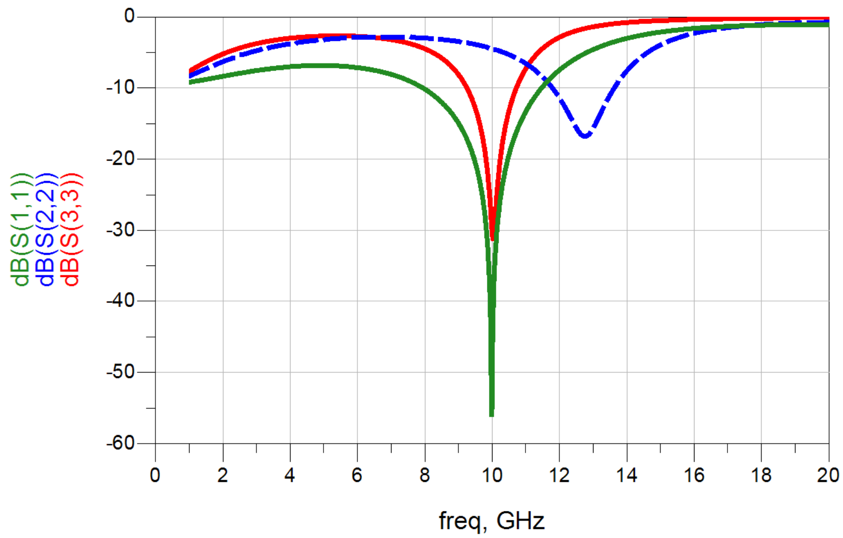

Figure F.25: Simulation results (reflection coefficients)

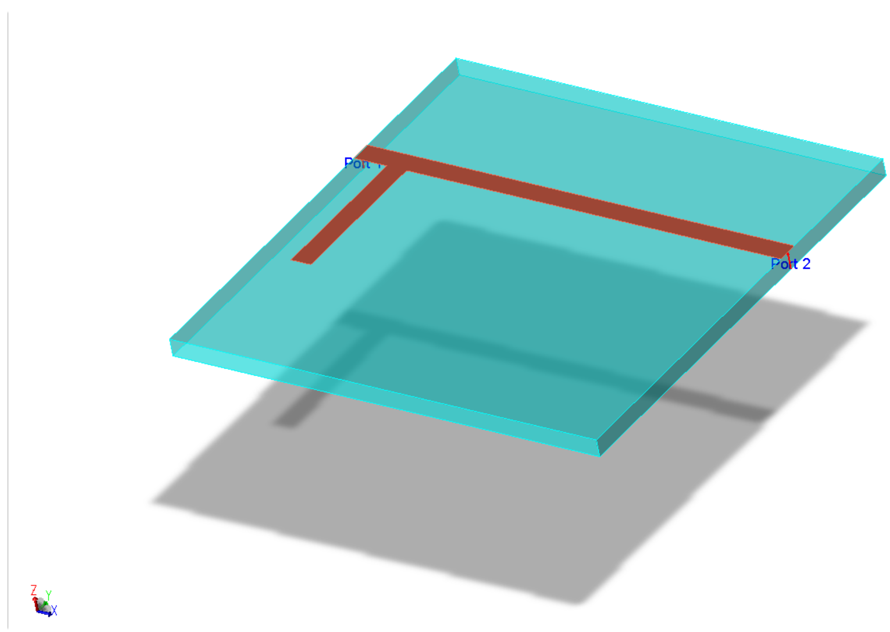

Figure F.26: Microstrip layout (Port 1 = 50 Ω; Port 2 = load impedance)

F.6 Problem 6.6

The scattering matrix of a Wilkinson power divider is given as

$$\mathbf{S}_{\text{Wilkinson}} = \frac{-j}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{F.42}$$

The scattering matrix of a *lossless* component fulfils the unitary matrix condition: $\mathbf{S}^{\mathrm{T}}\mathbf{S}^* = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. We will show, that the scattering matrix of a Wilkinson power divider does not fulfil the unitary matrix condition.

The Wilkinson power divider is reciprocal ($s_{ij} = s_{ji}$), so we write

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{pmatrix} \begin{pmatrix} s_{11}^* & s_{12}^* & s_{13}^* \\ s_{12}^* & s_{22}^* & s_{23}^* \\ s_{13}^* & s_{23}^* & s_{33}^* \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{F.43}$$

The first equation we derive from the matrix is

$$|s_{11}|^2 + |s_{12}|^2 + |s_{13}|^2 = 1 \tag{F.44}$$

Putting in the numbers from the scattering matrix of the Wilkinson divider yields

$$|0|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} + \frac{1}{2} = 1 \tag{F.45}$$

The next equation we get reads

$$\underbrace{s_{11}}_{=0} s_{12}^* + s_{12} \underbrace{s_{22}^*}_{=0} + s_{13}s_{23}^* = 0 \tag{F.46}$$

We end up with

$$s_{13}s_{23}^* = \left(\frac{-j}{\sqrt{2}}\right)\left(\frac{j}{\sqrt{2}}\right) \neq 0 \tag{F.47}$$

The equation is not satisfied. Therefore, the scattering matrix of the Wilkinson divider is no unitary matrix.

F.7 Problem 6.7

The design formulas for a branchline coupler with unequal power split are given in Section 6.8.2. A power ratio of 3:1 means that 75% of the incoming power P_{a1} at port 1 is transferred to port 2 and 25% is transferred to port 3.

The value of k is

$$k = \left|\frac{s_{31}}{s_{21}}\right| = \sqrt{\frac{P_{\mathrm{b3}}/P_{\mathrm{a1}}}{P_{\mathrm{b2}}/P_{\mathrm{a1}}}} = \sqrt{\frac{0.25}{0.75}} = \frac{1}{\sqrt{3}} \tag{F.48}$$

where

$$P_{\mathrm{b2}} = |s_{21}|^2 P_{\mathrm{a1}} \qquad \text{and} \qquad P_{\mathrm{b3}} = |s_{31}|^2 P_{\mathrm{a1}} \tag{F.49}$$

The absolute values of the transmission coefficients are

$$|s_{21}| = \sqrt{0.75} = -1.249\,\mathrm{dB} \qquad \text{and} \qquad |s_{31}| = \sqrt{0.25} = -6.021\,\mathrm{dB} \tag{F.50}$$

All port reference impedances are equal to $50\,\Omega$:

$$Z_{01} = Z_{02} = Z_0 = 50\,\Omega \tag{F.51}$$

So, the line impedances become

$$Z_{0\mathrm{a}} = \frac{Z_{01}}{k} = 86.6\,\Omega \tag{F.52}$$

$$Z_{0\mathrm{b}} = \frac{Z_0}{\sqrt{1+k^2}} = 43.3\,\Omega \tag{F.53}$$

$$Z_{0\mathrm{c}} = \frac{Z_{02}}{k} = 86.6\,\Omega \tag{F.54}$$

Each line length corresponds to a quarter wave length at $f = 6\,\mathrm{GHz}$ (electrical line lenth is 90°). Figure F.27 shows a circuit simulation using ideal lines. The corresponding scattering parameters are given in Figure F.28.

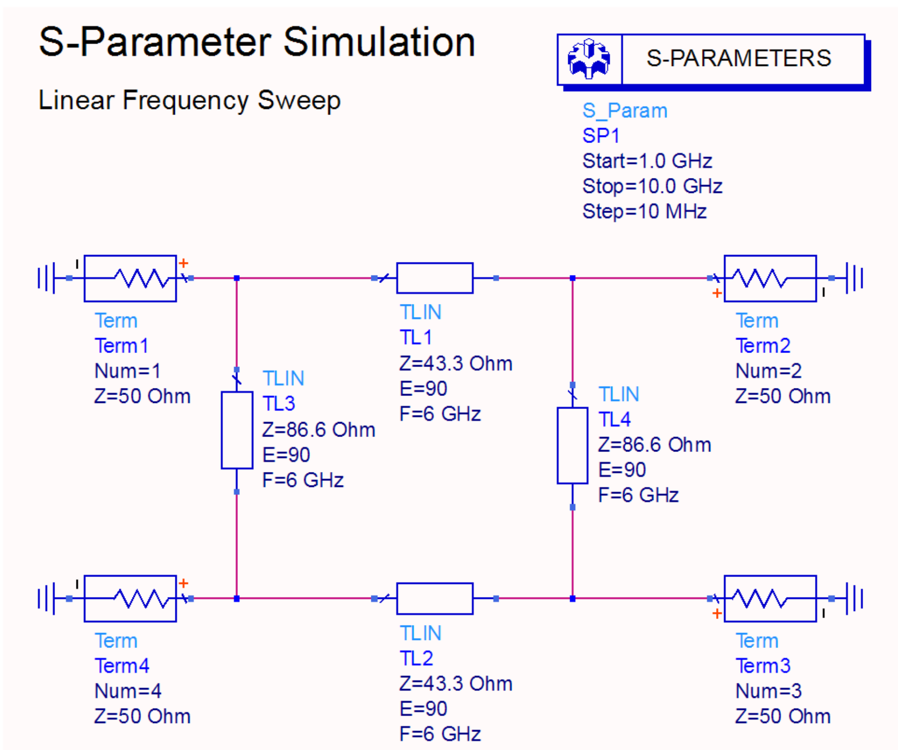

Figure F.27: Unequal split branchline coupler with ideal lines

Finally, we will look at a design with microstrip lines. Choosing substrate parameters ($h = 635\,\mu\mathrm{m}$ and $\varepsilon_\mathrm{r} = 9.8$) we can calculate line lengths and widths with TX-Line.

$$w_\mathrm{a} = w_\mathrm{c} = 0.140\,\mathrm{mm} \qquad \text{and} \qquad \ell_\mathrm{a} = \ell_\mathrm{c} = 5.066\,\mathrm{mm} \tag{F.55}$$

$$w_\mathrm{b} = 0.814\,\mathrm{mm} \qquad \text{and} \qquad \ell_\mathrm{b} = 4.75\,\mathrm{mm} \tag{F.56}$$

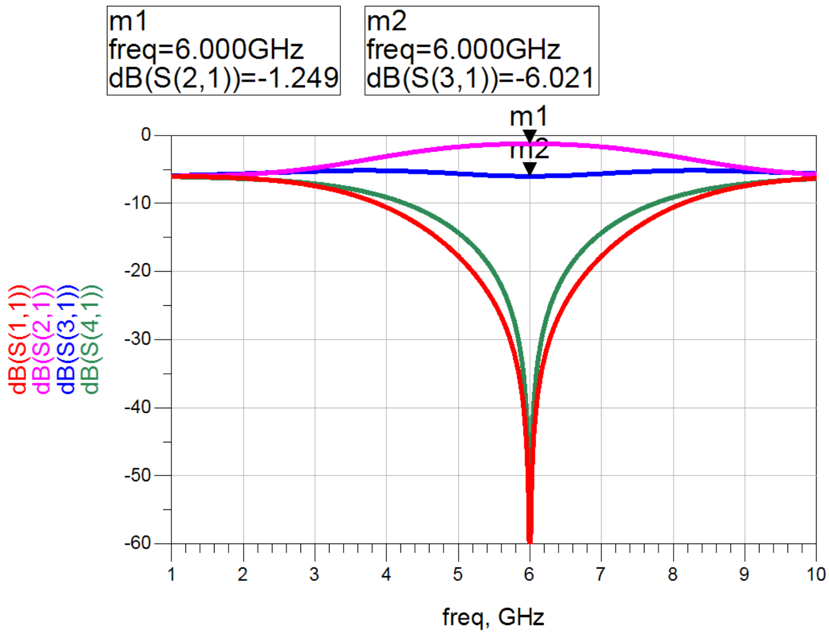

Figure F.28: Scattering parameters of unequal split branchline coupler with ideal lines

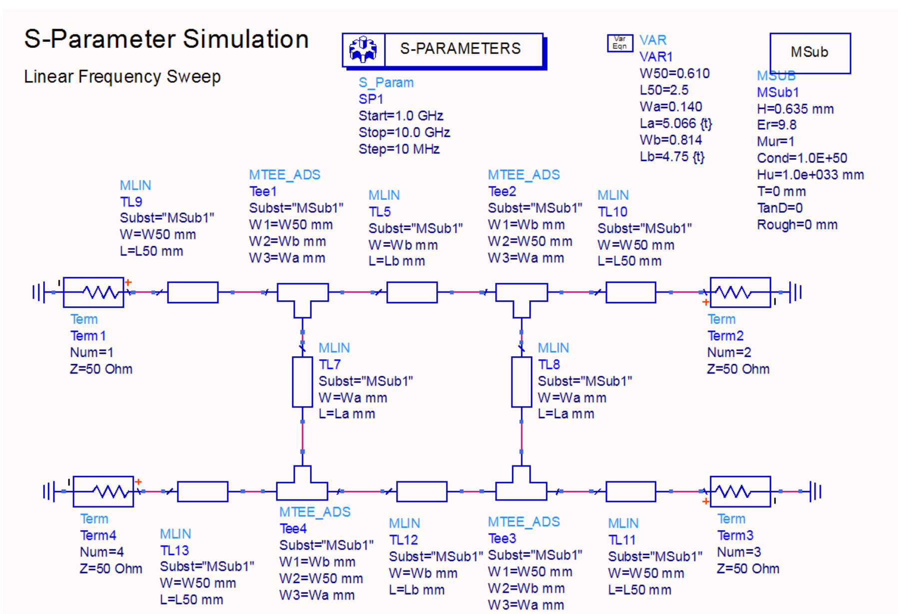

Figure F.29: Unequal split branchline coupler with microstrip lines

Figure F.29 shows a circuit simulation using microstrip lines. The corresponding scattering parameters are given in Figure F.30. The layout is shown in Figure F.31.

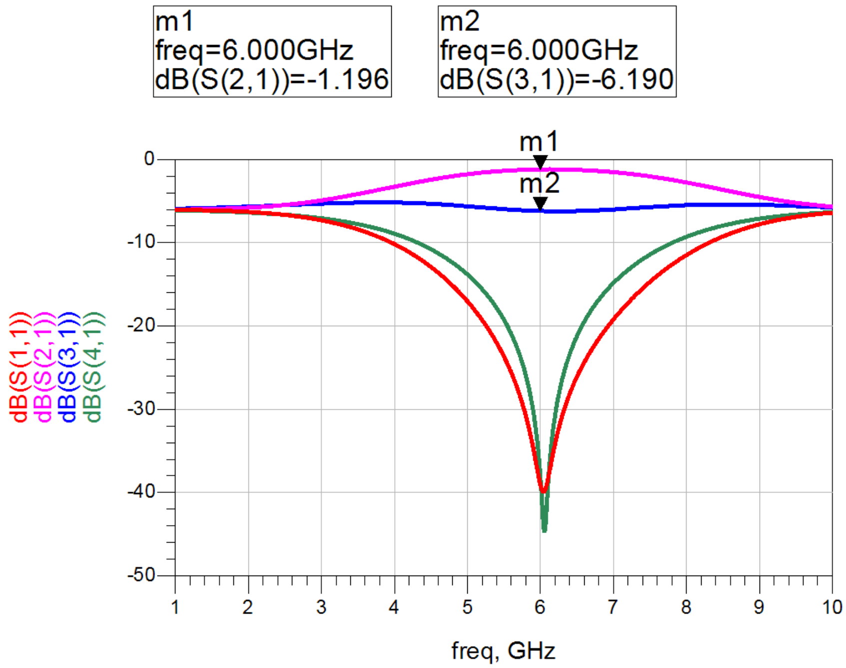

Figure F.30: Scattering parameters of unequal split branchline coupler with microstrip lines

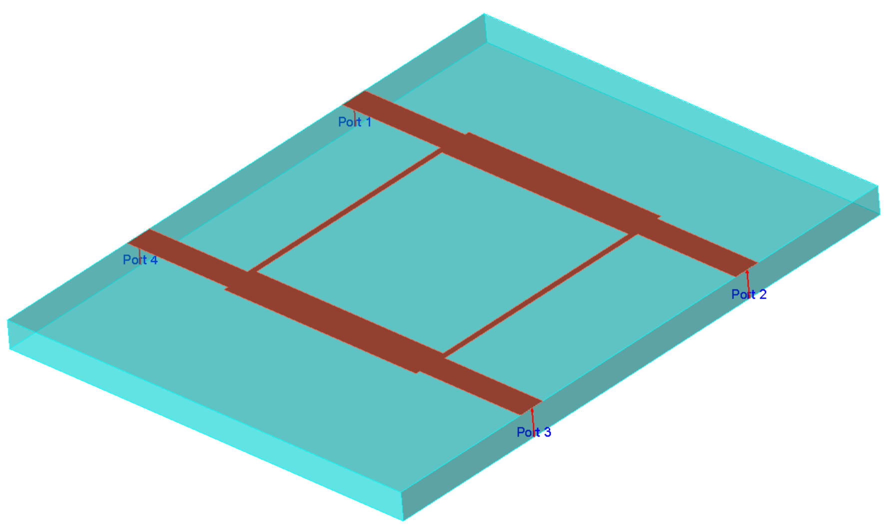

Figure F.31: Layout of unequal split branchline coupler with microstrip lines

F.8 Problem 6.8

We follow the design procedure described in Section 6.4.2.1. The order $n = 3$ is derived from Table 6.2 (book page 205) in order to achieve the desired attenuation at twice the cut-off frequency. The Butterworth filter coefficients are given by Equation 6.26.

$$a_1 = 1 \qquad a_2 = 2 \qquad a_3 = 1 \tag{F.57}$$

Our first design starts with a shunt capacitance. We get

$$C_1 = 2.27\,\mathrm{pF} \qquad L_2 = 45.47\,\mathrm{nF} \qquad C_3 = C_1 \tag{F.58}$$

The second design start with a series inductance. The component values are

$$L_1 = 22.74\,\mathrm{nH} \qquad C_2 = 4.547\,\mathrm{pF} \qquad L_3 = L_1 \tag{F.59}$$

Figure F.32 shows the schematics and Figure F.33 displays the transmission coefficients. For comparison we included an automatic design guide (a software assistant). The response type "maximally flat" indicates a Butterworth filter. Figure F.34 shows the circuit automatically designed by ADS design guide. The circuit corresponds to our manual design.

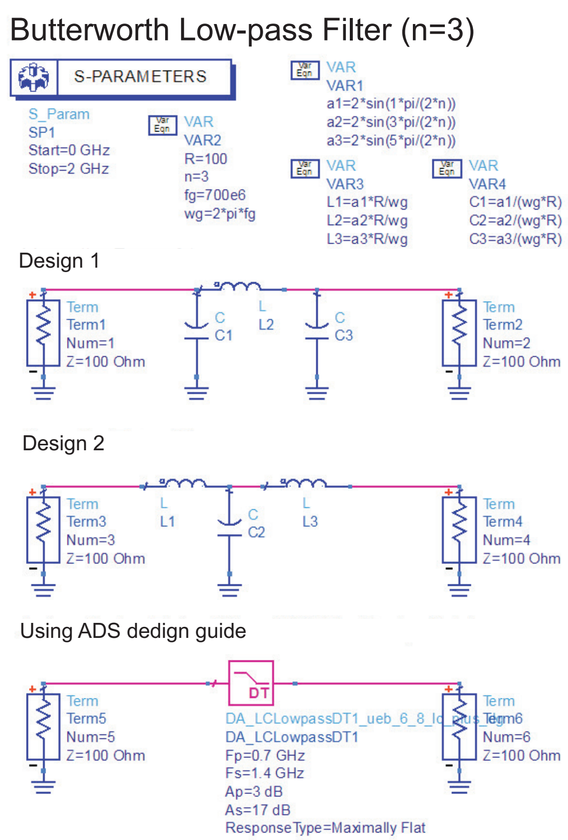

Figure F.32: Butterworth Filter

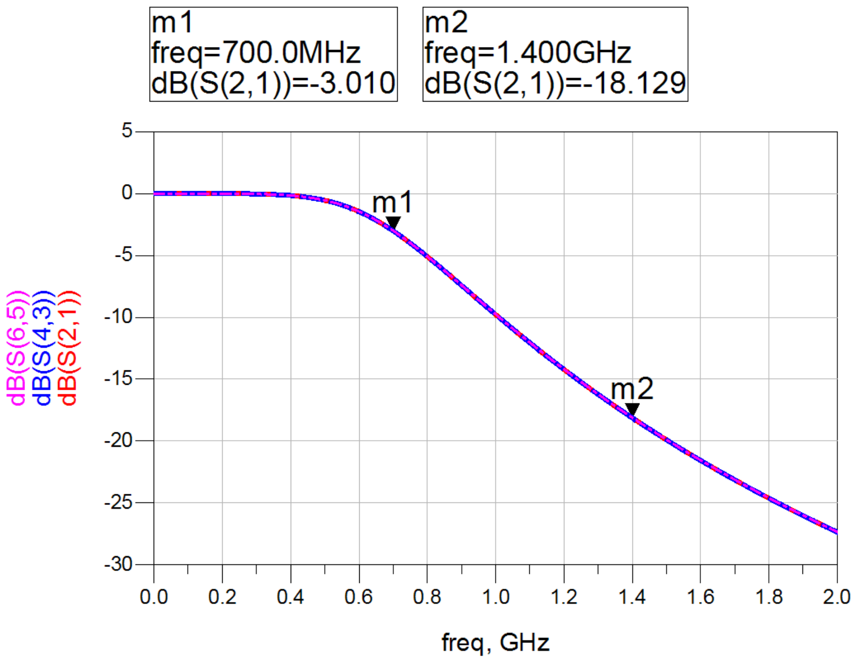

Figure F.33: Transmission coefficient of Butterworth Filter

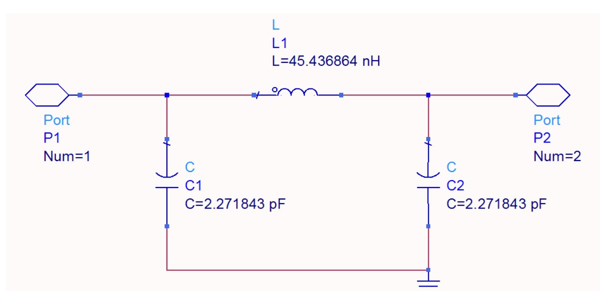

Figure F.34: Circuit automatically created by ADS design guide

F.9 Problem 6.9

A band-pass filter can be derived from a low-pass prototype (Section 6.4.2.3, book page 210). Equation 6.33 yields

$$\frac{f_{\mathrm{s,LP}}}{f_{\mathrm{c}}} = 2\frac{f_{\mathrm{s2}} - f_0}{f_{\mathrm{p2}} - f_{\mathrm{p1}}} = 3 \tag{F.60}$$

The stop frequency $f_{\mathrm{s,LP}}$ of the corresponding low-pass prototype filter is tree-times the value of the cut-off frequency f_{c}. Table 6.2 (book page 205) only lists attenuation values of up to two-fold values of the cut-off frequency. Therefore, we apply Equation 6.25 to determine the necessary filter order: An order of $n = 3$ yields an attenuation of 28.6 dB.

The Butterworth filter coefficients are given by Equation 6.26.

$$a_1 = 1 \qquad a_2 = 2 \qquad a_3 = 1 \tag{F.61}$$

Furthermore, we get

$$a_0 = 1 \qquad a_4 = 1 \tag{F.62}$$

Using Equations 6.30 to 6.32 yields

$$f_0 = 8.25\,\mathrm{GHz} \qquad \text{and} \qquad \omega_0 = 51.836\,\mathrm{GHz} \tag{F.63}$$

$$B = 0.5\,\mathrm{GHz} \qquad \text{and} \qquad BW = 3.1416\,\mathrm{GHz} \tag{F.64}$$

Next, we calculate the auxiliary values

$$J_{01} = J_{34} = 6.171 \cdot 10^{-3}\frac{1}{\Omega} \quad \text{and} \quad J_{12} = J_{23} = 1.346 \cdot 10^{-3}\frac{1}{\Omega} \tag{F.65}$$

The even-mode and odd-mode impedances are listed in Table F.3. With TX-Line we determine the line widths w, line lengths ℓ and distances (spacing) s between the lines.

a_i	Z_{0e}/Ω	Z_{0e}/Ω	w/μm	s/μm
$a_0 = a_4 = 1$	$Z_{0e,1} = Z_{0e,4} = 70.19$	$Z_{0e,1} = Z_{0e,4} = 39.33$	$w_1 = w_4 = 363.2$	$s_1 = s_4 = 161.4$
$a_1 = a_3 = 1$	$Z_{0e,2} = Z_{0e,3} = 53.59$	$Z_{0e,2} = Z_{0e,3} = 46.86$	$w_2 = w_3 = 456.4$	$s_2 = s_3 = 778.5$
$a_2 = 2$				

Table F.3: Filter parameters and dimensions

The line length do not consider open-end and coupling effects. Therefore, we introduce a term to tune the line lengths in order to achieve the desired center frequency f_0. ADS provides sliders to perform this design step interactively. The final circuit is given in Figure F.35. The transmission coefficient in Figure F.36 shows good agreement with the specifications. The layout is given Figure F.37.

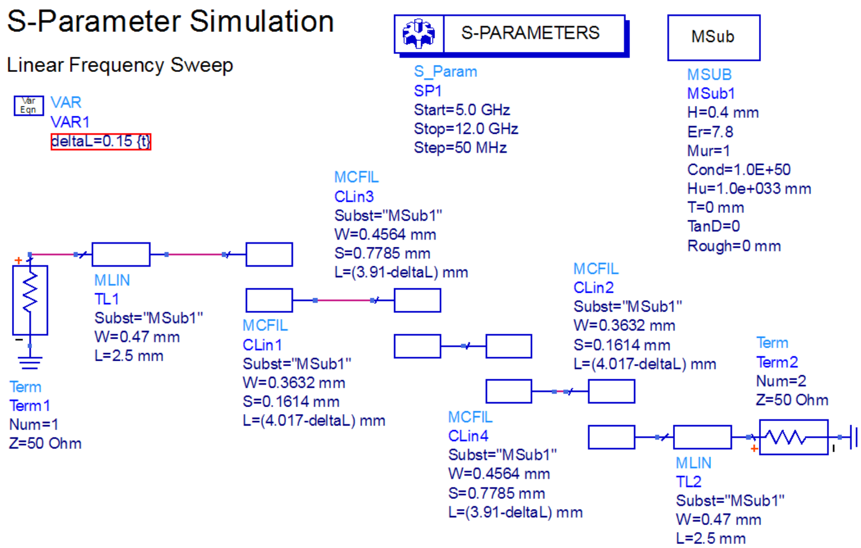

Figure F.35: Schematic of microstrip band-pass filter

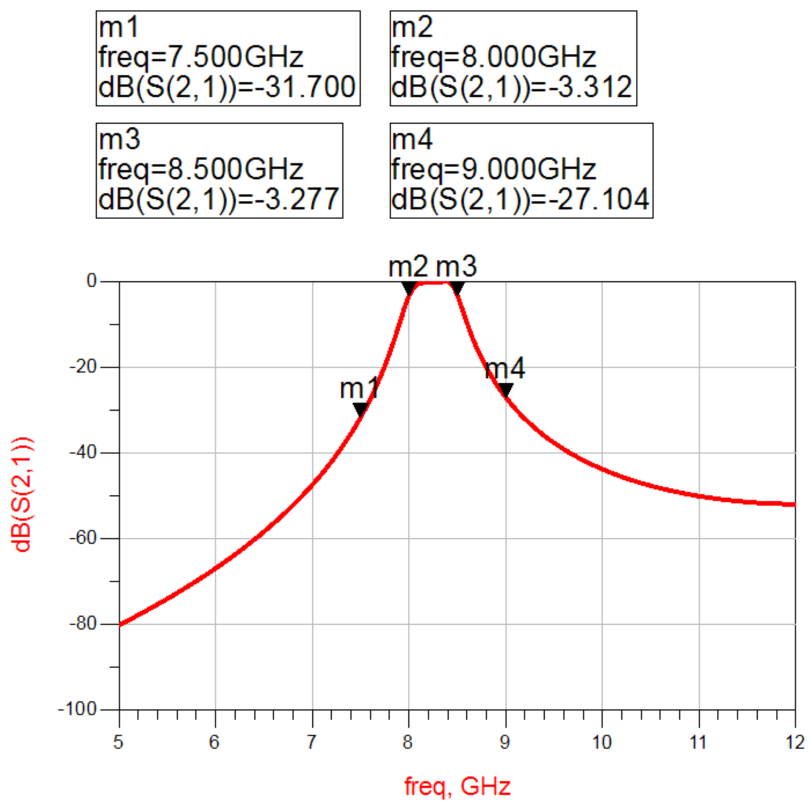

Figure F.36: Transmission coefficient of microstrip band-pass filter

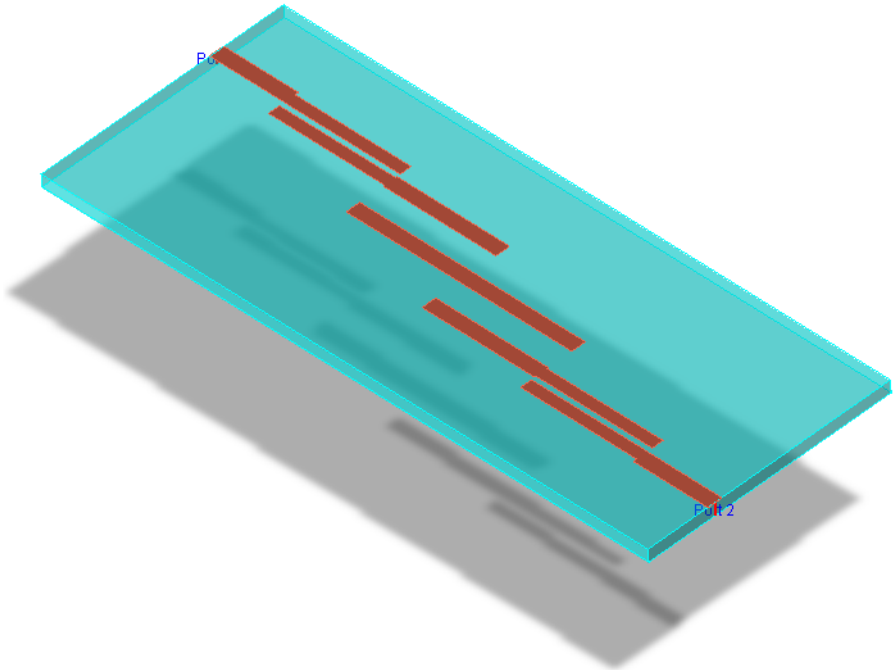

Figure F.37: Layout of microstrip band-pass filter

(Last modified: 26.02.2013)